

## Precalc Warm Up # 2-1

1. Solve:  $3^{2x-1} = 48$

2. In an account that pays 4.6% annual interest rate compounded monthly, how long would it take to double your investment? What if it were compounded continuously?

## HW Questions: p. 276

In Exercises 1–20, use the properties of logarithms to write the expression as a sum, difference, or multiple of logarithms.

1.  $\log_2 5x$

3.  $\log_3 \frac{5}{x}$

5.  $\log_8 x^4$

7.  $\ln \sqrt{z}$

9.  $\log_2 xyz$

11.  $\ln \sqrt{a-1}$

13.  $\ln z(z-1)^2$

15.  $\log_b \frac{x^2}{y^2 z^3} = \log_b x^2 - \log_b y^2 z^3$

$$= 2 \log_b x - (\log_b y^2 + \log_b z^3)$$

$$2 \log_b x - 2 \log_b y - 3 \log_b z$$

17.  $\ln \sqrt[3]{x/y}$

19.  $\log_9 \frac{x^4 \sqrt{y}}{z^5}$

In Exercises 21–40, write the expression as the logarithm of a single quantity.

21.  $\ln x + \ln 2$

23.  $\log_4 z - \log_4 y$

25.  $2 \log_2(x + 4)$

27.  $\ln x - 3 \ln(x + 1)$

29.  $\frac{1}{3} \log_3 5x$

31.  $\log_3(x - 2) - \log_3(x + 2)$

33.  $\ln x - 2[\ln(x + 2) + \ln(x - 2)]$

35.  $\frac{1}{3}[2 \ln(x + 3) + \ln x - \ln(x^2 - 1)]$

37.  $\frac{1}{3}[\ln y + 2 \ln(y + 4)] - \ln(y - 1)$

39.  $2 \ln 3 - \frac{1}{2} \ln(x^2 + 1)$

In Exercises 41–56, approximate the logarithm using the properties of logarithms, given  $\log_b 2 \approx 0.3562$ ,  $\log_b 3 \approx 0.5646$ , and  $\log_b 5 \approx 0.8271$ .

41.  $\log_b 6$

47.  $\log_b \sqrt{2}$

$$\log_b 2^{1/2}$$

$$\frac{1}{2}(\log_b 2)$$

$$\frac{1}{2}(0.3562)$$

51.  $\log_b \left(\frac{1}{4}\right)$

55.  $\log_b \frac{(4.5)^3}{\sqrt{3}}$

$$\log_b 2^{-2}$$

$$3\log_b(4.5) - \frac{1}{2}\log_b 3$$

$$\frac{9}{2}$$

$$3\log_b\left(\frac{3^2}{2}\right) - \frac{1}{2}\log_b 3$$

$$3[\log_b 3^2 - \log_b 2] - \frac{1}{2}(0.5646)$$

$$3[2\log_b 3 - (0.3562)] - \frac{1}{2}(0.5646)$$

In Exercises 57–62, find the exact value of the logarithm.

59.  $\log_4 16^{1.2}$

$$1.2(\log_4 16)$$

$$1.2(\log_4 4^2)$$

$$1.2(2)$$

$$\boxed{2.4}$$

In Exercises 63–70, use the properties of logarithms to simplify the given logarithmic expression.

63.  $\log_4 8$

$$\begin{aligned} \log_4(4 \cdot 2) \\ \log_4 4 + \log_4 2 \\ 1 + \frac{1}{2} \\ \boxed{\frac{3}{2}} \end{aligned}$$

67.  $\log_5\left(\frac{1}{250}\right)$

$$\begin{aligned} \log_5 \frac{1}{(125)(2)} \\ \log_5 [5^{-3}(2^{-1})] \\ \log_5 5^{-3} + \log_5 2^{-1} \\ -3 - \log_5 2 \end{aligned}$$

71. Prove that  $\log_b \frac{u}{v} = \log_b u - \log_b v$ .

Tools

A logarithm is an exponent.

Definition:  $y = \log_a x$  iff  $x = a^y$

$a$  must be positive and  $\neq 1$ , and  $x$  must be positive

Basic Log Properties:

$$\log_a a = \quad \log_a 1 = \quad \log_a a^x =$$

Do logs distribute?

$$\log_2 2 + \log_2 8 \stackrel{?}{=} \log_2 (2 + 8)$$

$$\log_2 2 + \log_2 8 = \log_2 (2 \cdot 8) \quad \leftarrow \text{log rule \#1}$$

$$a^{m+n} = a^m \cdot a^n$$

log rule #1:

$$\log_a uv = \log_a u + \log_a v$$

log rule #2:

$$\log_a(u/v) = \log_a u - \log_a v$$

log rule #3:

$$\log_a u^b = b \log_a u$$

Solving Logarithmic & Exponential Equations:

Equate bases if possible:

$$\text{Ex: } 2^{x+1} = \frac{1}{32}$$

$$\begin{aligned} 2^{x+1} &= 2^{-5} \\ x+1 &= -5 \\ x &= -6 \end{aligned}$$

$$\text{Ex: } 9^{x+2} = \frac{1}{27}$$

$$\begin{aligned} 3^{2x+4} &= 3^{-3} \\ 2x+4 &= -3 \\ x &= -\frac{7}{2} \end{aligned}$$

When you can't equate the bases, and  $x$  is **only** in the exponent, take the log of both sides, and then use log rule #3 and algebra to isolate  $x$ .

$$\begin{aligned}
 2^{x+1} &= 15^{2-x} \\
 (x+1)\ln 2 &= (2-x)\ln 15 \\
 x\ln 2 + \ln 2 &= 2\ln 15 - x\ln 15 \\
 x\ln 2 + x\ln 15 &= 2\ln 15 - \ln 2 \\
 x(\ln 2 + \ln 15) &= \ln 225 - \ln 2 \\
 (\ln 2 + \ln 15) & \quad (\ln 2 + \ln 15) = \\
 x &\approx 1.39
 \end{aligned}$$

When there are logs on only one side of the equation, simplify as much as possible (condense), then rewrite equation in exponential form:

$$\begin{aligned}
 \log_2(x+5) - \log_2(x-2) &= 3 \\
 \log_2\left(\frac{x+5}{x-2}\right) &= 3 \\
 (x-2)2^3 &= \frac{x+5}{x-2} \cdot (x-2) \\
 8x - 16 &= x + 5 \\
 7x &= 21 \\
 \boxed{x=3}
 \end{aligned}$$

If logs are on both sides, simplify both sides (condense), then use logic and algebra to finish the problem.

$$\begin{aligned}
 2\ln x + \ln 3 &= \ln 5 + 2\ln(x-1) \\
 \ln x^2 + \ln 3 &= \ln 5 + \ln(x-1)^2 \\
 \ln 3x^2 &= \ln(5(x-1)^2) \\
 3x^2 &= 5(x^2 - 2x + 1) \\
 0 &= 2x^2 - 10x + 5 \\
 0 &= 2x^2 - 10x + 5 \\
 x &= \frac{10 \pm \sqrt{100 - 4(2)(5)}}{2(2)} \\
 \frac{10 \pm \sqrt{60}}{4} &\approx \frac{4.44}{0.564}
 \end{aligned}$$

You try:

$$1) \quad 3^x = 5^{4x-2}$$

$$\begin{aligned}
 x \ln 3 &= (4x-2) \ln 5 \\
 x \ln 3 &= 4x \ln 5 - 2 \ln 5 \\
 2 \ln 5 &= 4x \ln 5 - x \ln 3 \\
 \ln 25 &= \frac{x(4 \ln 5 - \ln 3)}{4 \ln 5 - \ln 3}
 \end{aligned}$$

$$x \approx 0.603$$

$$2) \quad \log_5 (x + 100) - \log_5 (x) = 1$$

$$\log_5 \left( \frac{x+100}{x} \right) = 1$$

$$x \cdot 5 = \frac{x+100}{x} \cdot x$$

$$5x = x + 100$$

$$4x = 100$$

$$x = 25$$

Solve. Answer as an improper fraction.

$$\log_2 x - \log_2 (x - 1) = 3 \log_2 4$$

$$\log_2 \frac{x}{x-1} = \log_2 4^3$$

$$x = \frac{64}{63}$$

And don't forget about factoring quadratics !

$$(\log x)^2 + 3 \log x - 10 = 0$$

$$\text{let } y = \log x$$

$$y^2 + 3y - 10 = 0$$

$$(y - 2)(y + 5) = 0$$

$$(\log x - 2)(\log x + 5) = 0$$

$$\log x - 2 = 0 \quad \log x + 5 = 0$$

$$\log_{10} x = 2$$

$$\log_{10} x = -5$$

$$10^2 = x$$

$$10^{-5} = x$$

$$x = 100, \frac{1}{100000}$$

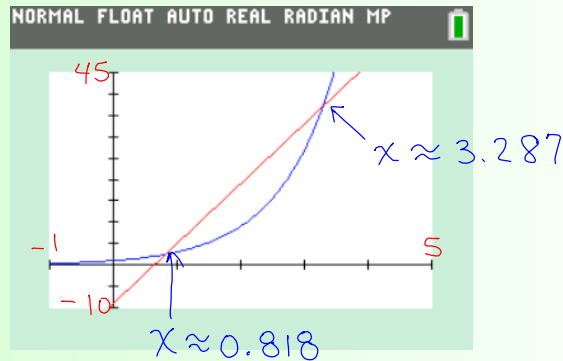


Finally, when  $x$  is in the exponent AND on the main line of the problem, graph it! There is no algebraic way to solve.

$$3^x = 14x - 9$$

Window

$$\begin{aligned} x\text{-min} &= -1 \\ x\text{-max} &= 5 \\ x\text{-scl} &= 1 \\ y\text{-min} &= -10 \\ y\text{-max} &= 45 \\ y\text{-scl} &= 5 \end{aligned}$$



Some last little tricks...

$$10^{\log 7} = 7 \quad \text{change to log form:} \\ 10^{\log_{10} 7} = 7 \quad \log_{10} x = \log_{10} 7$$

$$e^{\ln 8} = 8 \quad \text{like:}$$

$$2^{\log_2 7} = 7 \quad \log_2 2^x = x$$

$$5^{\log_5(x+7)} = x+7$$

HW: PC book

p. 284 #1-15 odd, 20,  
26, 49-63 odd, 65, 70

HW Quiz Tuesday

PC p. 259, 269, 276

\*Book to bring:  
Tues - Wed: SL book

Quiz Thurs

PC: 4.1- 4.4

SL: 7.1